School of Modern Optics

7 May 2013, Puebla, Mexico

Lecture 2

Liquid crystals and optical singularities

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Singular Optics & Liquid Crystals group

www.loma.cnrs.fr/spip.php?rubrique331

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Outline

1. Introduction to singular optics

2. Optical vortex generation

3. Spin-orbit interaction of light

4. Liquid crystal spin-to-orbital angular momentum converters

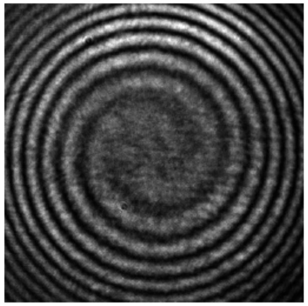
5. Towards integrated spin-orbit optical vortex generators

$$\mathbf{E} = \boxed{E}\,\boxed{e^{i\psi}}\,e^{-i\omega t}\,\boxed{\mathbf{e}}$$

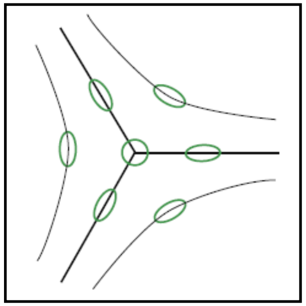
Different kinds of singularities in optics

Intensity

Phase



Polarization

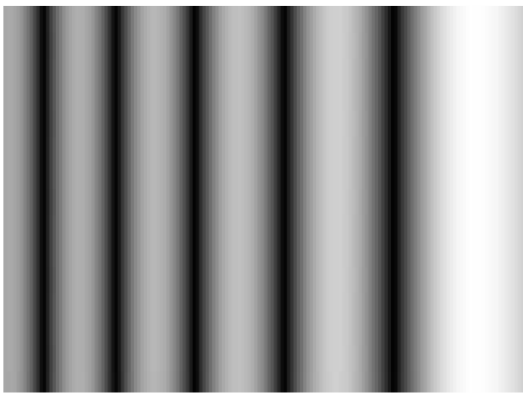


Topological defects of light : an old story

« Three wave singularities from the miraculous 1830s »

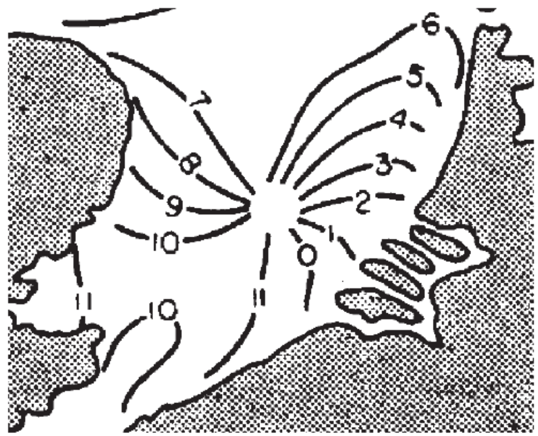
M. Berry, Nature **403**, 21 (2000)

Intensity



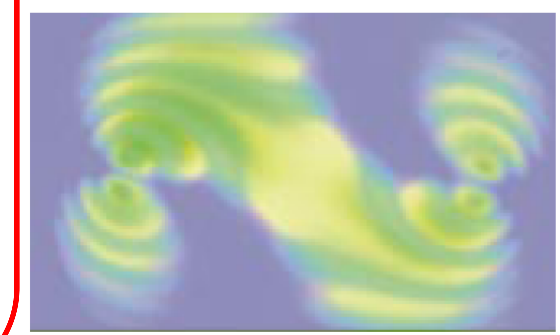
Airy's rainbow integral.

Phase



Whewell's amphidromy between England and Holland.

Polarization



Hamilton's diabolical points (bullseyes) in several square centimetres of overhead-projector transparency foil viewed obliquely through crossed polarizers; in each bullseye, the interference rings are contours of difference of wave speeds, centred on an optic axis, and the black stripes reflect geometric phases.

Let us consider coherent superposition of N plane waves

$$E(z=0,t) \;=\; \sum_{n=1}^{N} a_n e^{i[k_x^{(n)}x+k_y^{(n)}y]}$$

Optical phase singularities

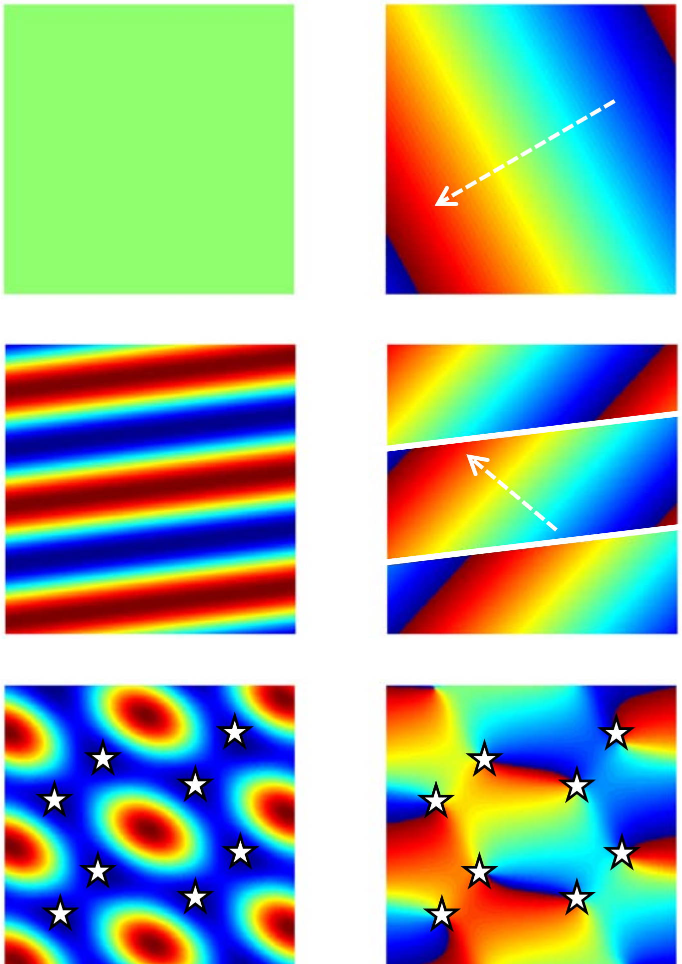
Let us consider coherent superposition of N plane waves

Intensity | Phase

N = 1

N = 2

N = 3

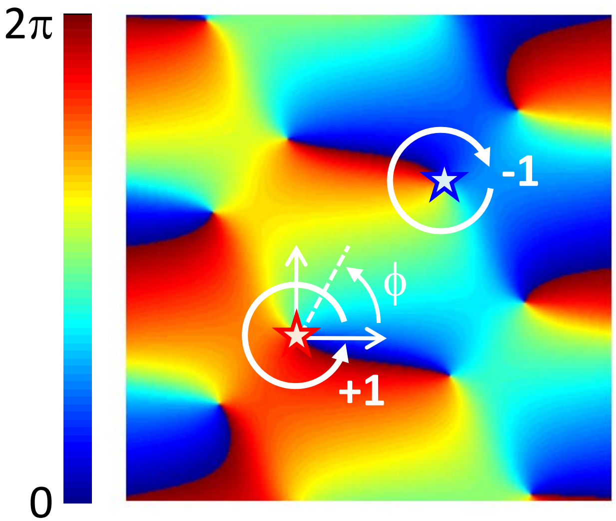


Uniform intensity & Phase ramp

Oscillating intensity & Phase ramp/steps

Patterned intensity & Phase singularities

Optical phase singularities

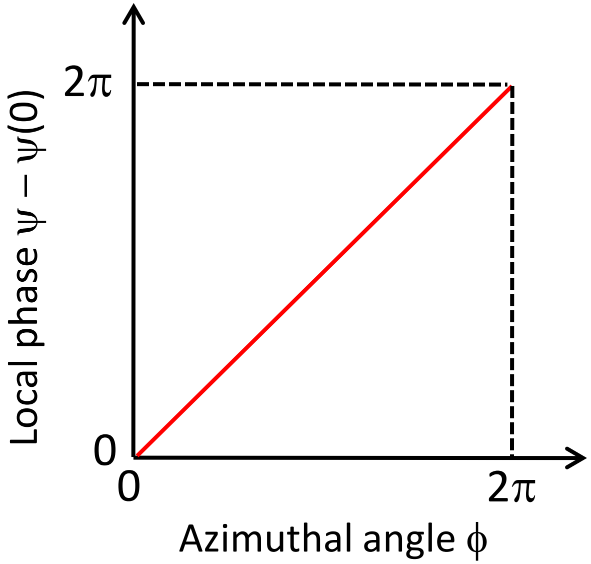


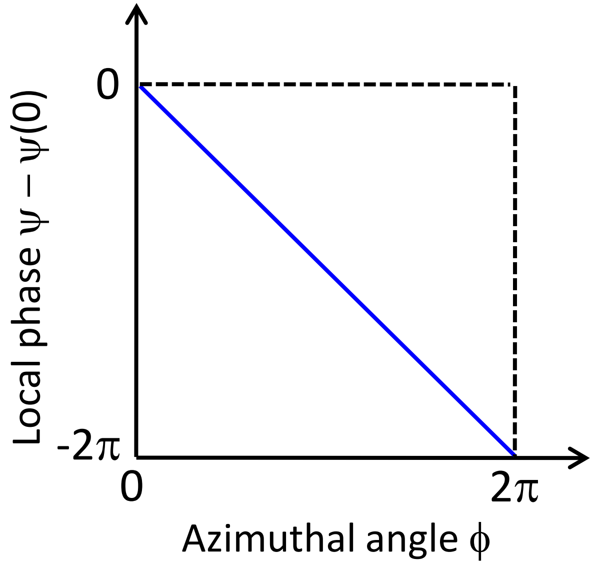
$$\mathbf{E} = E\ e^{i\psi}\ e^{-i\omega t}\ \mathbf{e}$$

Local azimuthal phase behavior is generic

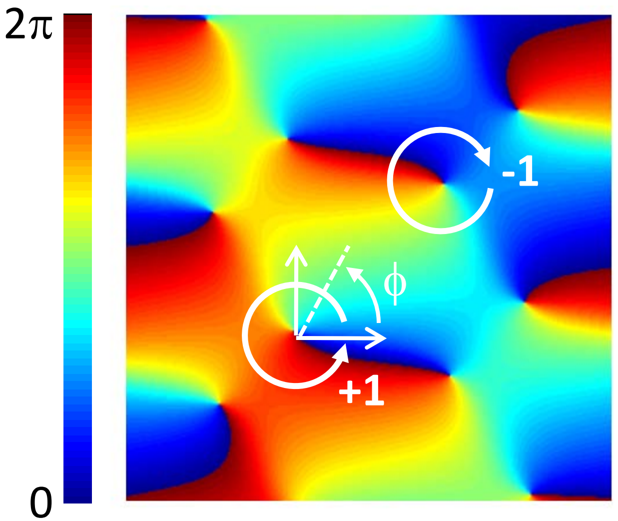
$$\psi = \pm\phi + \phi_0$$

Topological charge ±1





Optical phase singularities



$$\mathbf{E} = E\ e^{i\psi}\ e^{-i\omega t}\ \mathbf{e}$$

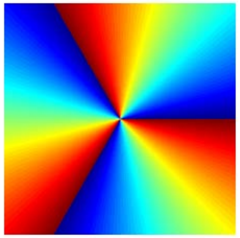
Local azimuthal phase behavior is generic

$$\psi = \pm\phi + \phi_0$$

Topological charge ± 1

Generalization to higher-order optical phase singularities

$\psi = \ell\phi$ with ℓ integer

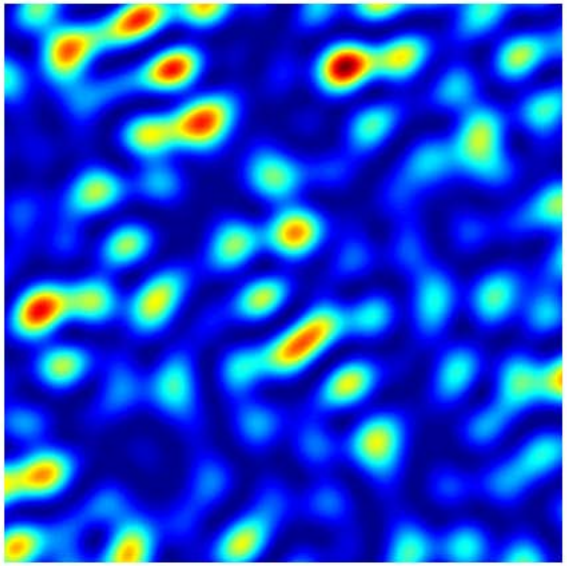


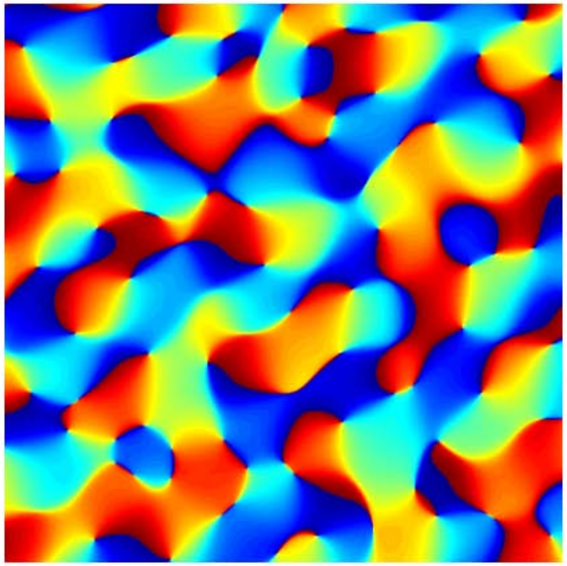
Example: $\ell = -3$

$$E(z=0,t) = \sum_{n=1}^{N} a_n e^{i[k_x^{(n)}x + k_y^{(n)}y]} \xrightarrow[N>>1]{} \text{Speckle field}$$

Intensity **Phase**

N = 10



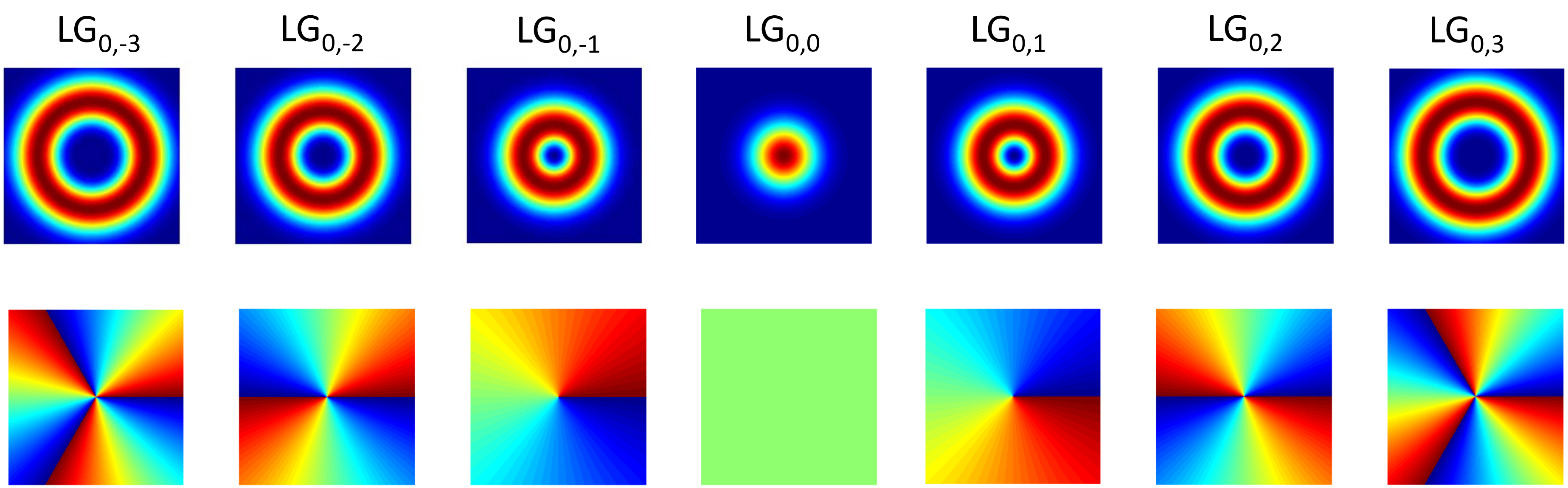


Speckle field : Collection of phase singularities with unit topological charge

What about usual beams ?

Laguerre-Gauss modes

$$LG_{p\ell} = \sqrt{\frac{2p!}{\pi\,(p+|\ell|)!}}\frac{1}{w(z)}\overbrace{\left[\frac{r\sqrt{2}}{w(z)}\right]^{|\ell|}\exp\left[\frac{-r^2}{w^2(z)}\right]L_p^{|\ell|}\left(\frac{2r^2}{w^2(z)}\right)}^{\text{Amplitude}}\underbrace{\exp[i\ell\phi]}_{\text{Spiraling phase}}\overbrace{\exp\left[\frac{ik_0 r^2 z}{2(z^2+z_R^2)}\right]}^{\text{Curvature of phase}}\overbrace{\exp\left[-i(2p+|\ell|+1)\tan^{-1}\left(\frac{z}{z_R}\right)\right]}^{\text{Gouy phase}}$$



Optical phase singularities : how to identify the lab ?

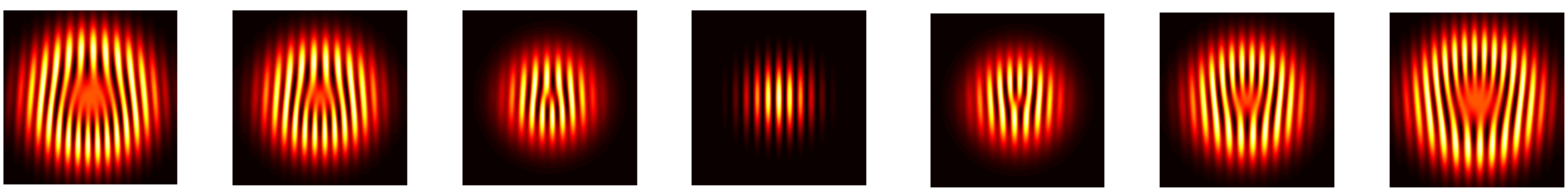
Intensity patterns $\left|LG_{0,\ell}\right|^2$

ℓ = -3 | ℓ = -2 | ℓ = -1 | ℓ = 0 | ℓ = 1 | ℓ = 2 | ℓ = 3

Interference fringes $\left|LG_{0,0}+LG_{0,\ell}\right|^2$

$LG_{0,\ell}$

$LG_{0,0}$



ℓ-forks

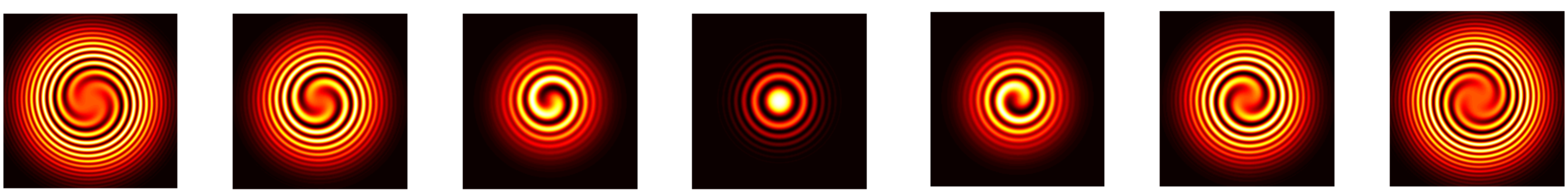
Intensity patterns $\left|LG_{0,\ell}\right|^2$

ℓ = -3 | ℓ = -2 | ℓ = -1 | ℓ = 0 | ℓ = 1 | ℓ = 2 | ℓ = 3

Interference fringes $\left|LG_{0,0} + LG_{0,\ell}\right|^2$

$LG_{0,\ell} \longrightarrow$

$LG_{0,0} \longrightarrow$



ℓ-arm spiral

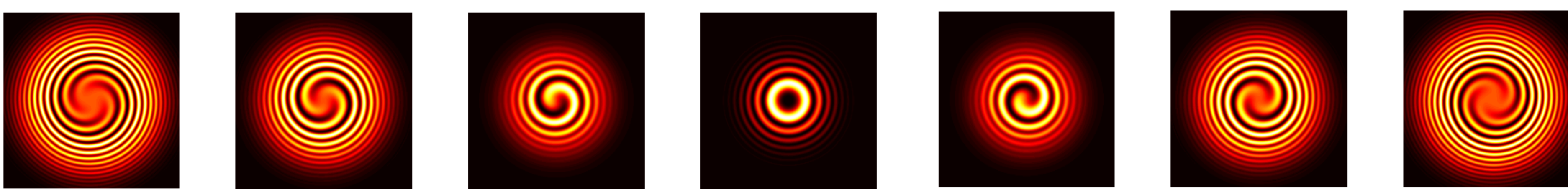
Optical phase singularities : how to identify the lab ?

Intensity patterns $\left|LG_{0,\ell}\right|^2$

$\ell = -3$ | $\ell = -2$ | $\ell = -1$ | $\ell = 0$ | $\ell = 1$ | $\ell = 2$ | $\ell = 3$

Interference fringes $\left|LG_{0,0} - LG_{0,\ell}\right|^2$

$$LG_{0,\ell} \longrightarrow$$
$$e^{i\pi} LG_{0,0} \longrightarrow$$



Dark spot $\neq$ Optical vortex

A basic feature of light fields

Per photon of a light beam

Energy	Linear momentum	Angular momentum
$E = \hbar\omega$	$\mathbf{p} = \hbar\mathbf{k}$	$j_z = s_z + l_z$

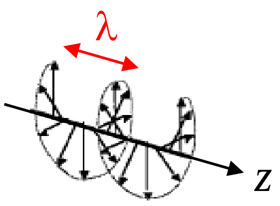
Per photon of a light beam

Energy	Linear momentum	Angular momentum
$E = \hbar\omega$	$\mathbf{p} = \hbar\mathbf{k}$	$j_z = s_z + l_z$

Spin angular momentum

Left/right-handed circular polarization state

$$s_z = \pm\hbar$$

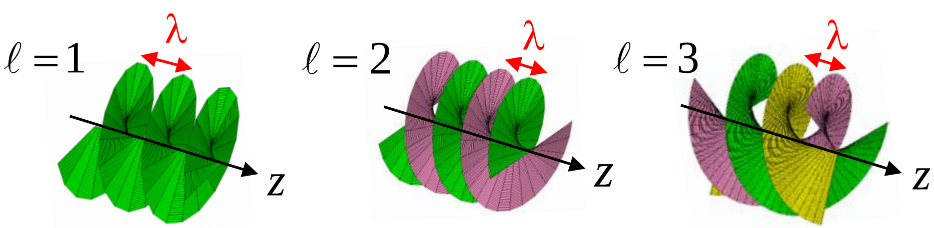


Electric field

Orbital angular momentum

Phase spatial distribution : $\exp(i\ell\varphi)$

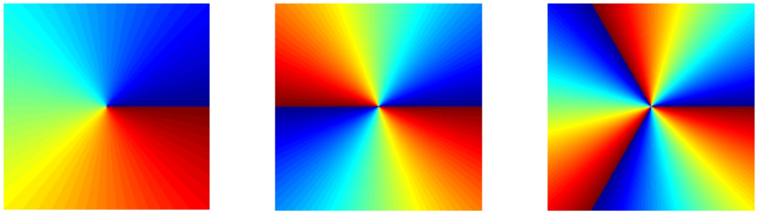
$$l_z = \ell\hbar$$



Equiphase surface

Optical phase singularities with topological charge ℓ

Phase reasons



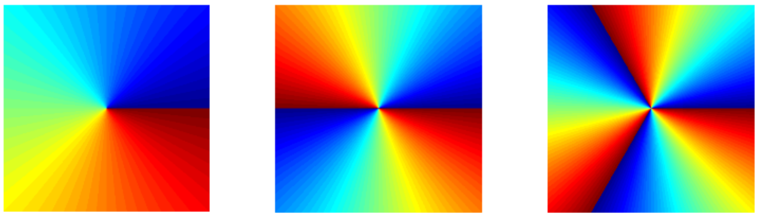
- **Rotational optomechanics**
  (torque $\ell\hbar$)

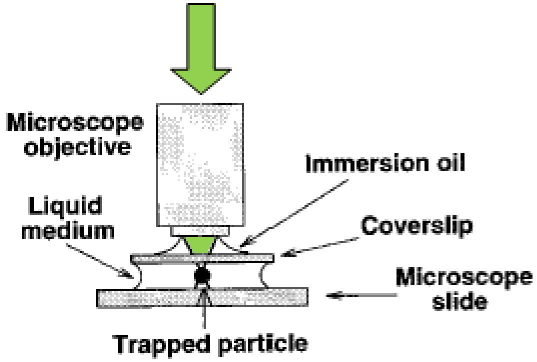
- **Optical information**
  (topological information ℓ)

- **Field topology**
  (singularity)

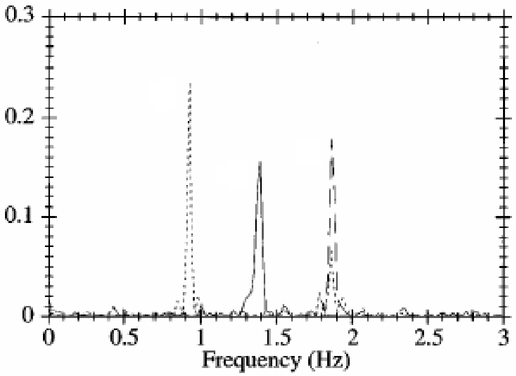
Controlling optical orbital angular momentum : why ?

Phase reasons





M. E. Friese *et al.*, PRA **54**, 1593 (1996)



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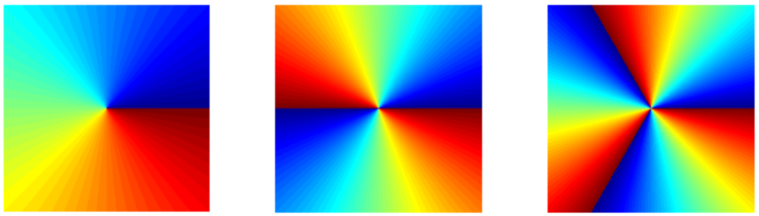

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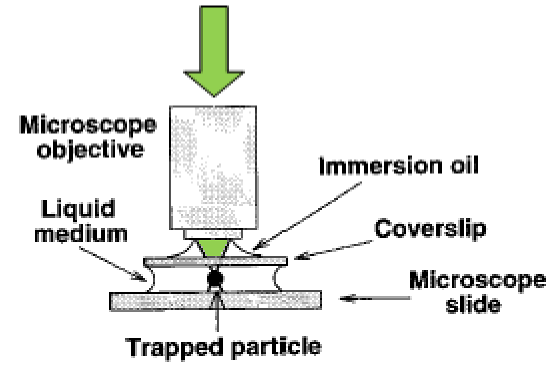

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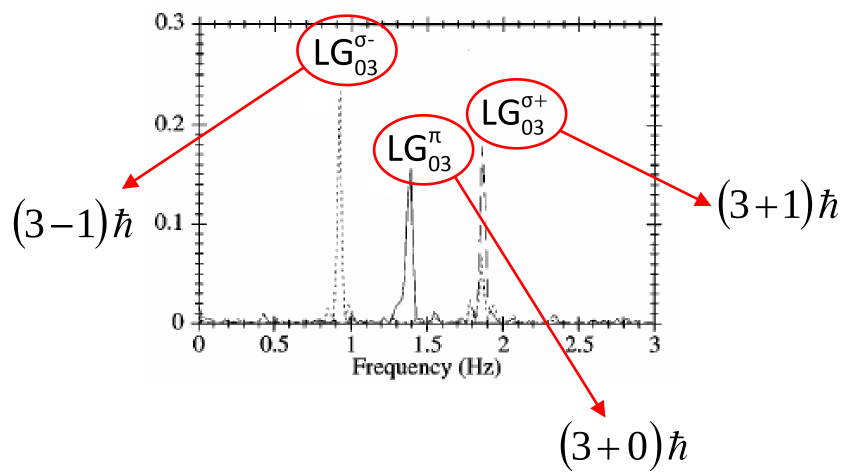
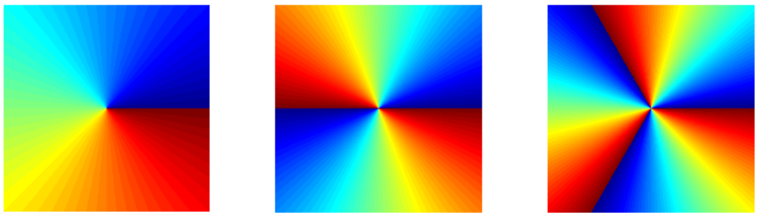


Transferred angular momentum per absorbed photon : (2, 3, 4)$\hbar$

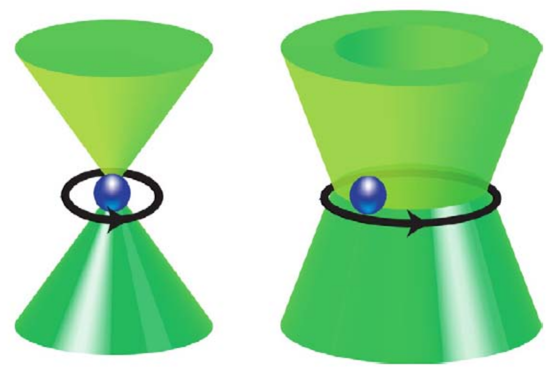
Controlling optical orbital angular momentum : why ?

Phase reasons



- **Rotational optomechanics** (torque $\ell\hbar$)
- **Optical information** (topological information ℓ)
- **Field topology** (singularity)

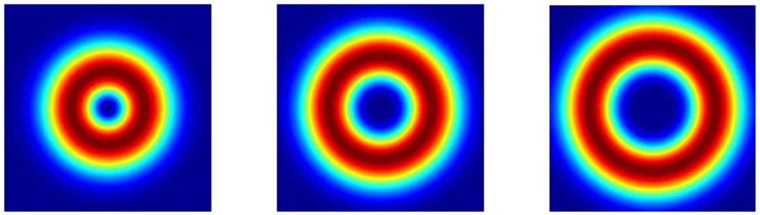
Distinct rotational modes



Spinning **Orbiting**

Controlling optical orbital angular momentum : why ?

Amplitude reasons

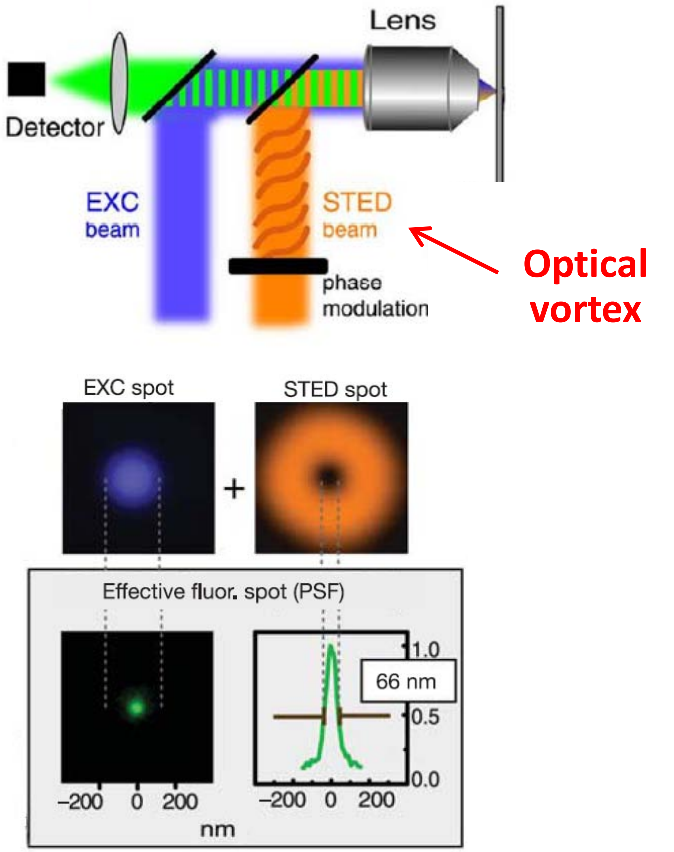


- **Unconventional trapping**
  (on-axis null intensity)

- **Super-resolution optical imaging**
  (STED microscopy)

- **Astronomical imaging**
  (vortex coronagraphy)

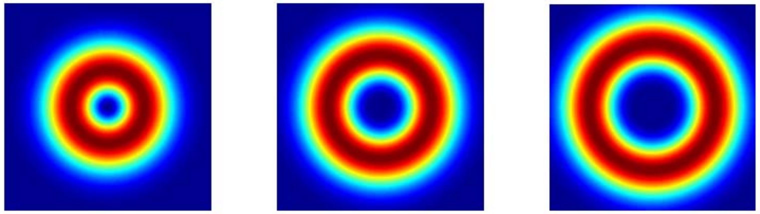
STimulated Emission Depletion



K. I. Willig *et al.*, Nature **440**, 935 (2006)

Controlling optical orbital angular momentum : why ?

Amplitude reasons



- **Unconventional trapping**
  (on-axis null intensity)

- **Super-resolution optical imaging**
  (STED microscopy)

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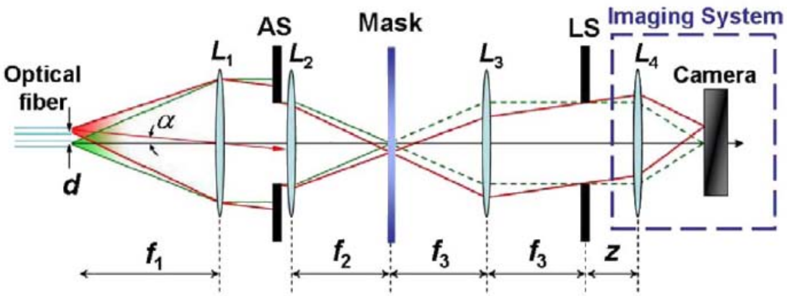
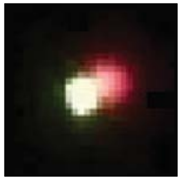
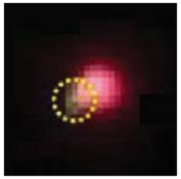
  (vortex coronagraphy)

Vortex coronagraph basic set-up





without vortex mask



with vortex mask

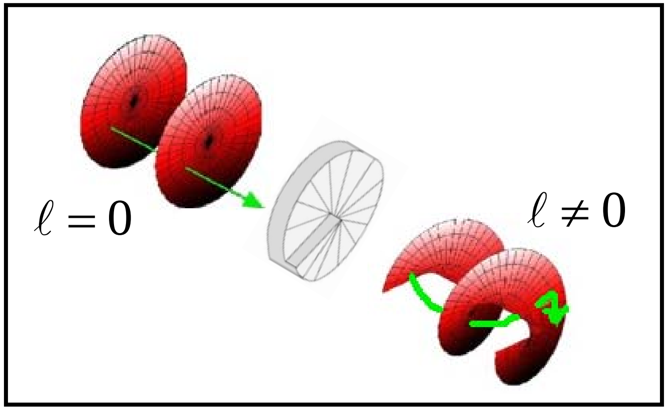
J. H. Lee *et al.*, PRL **97**, 053901 (2006)

Outline

1. Introduction to singular optics
2. **Optical vortex generation**
3. Spin-orbit interaction of light
4. Liquid crystal spin-to-orbital angular momentum converters
5. Towards integrated spin-orbit optical vortex generators

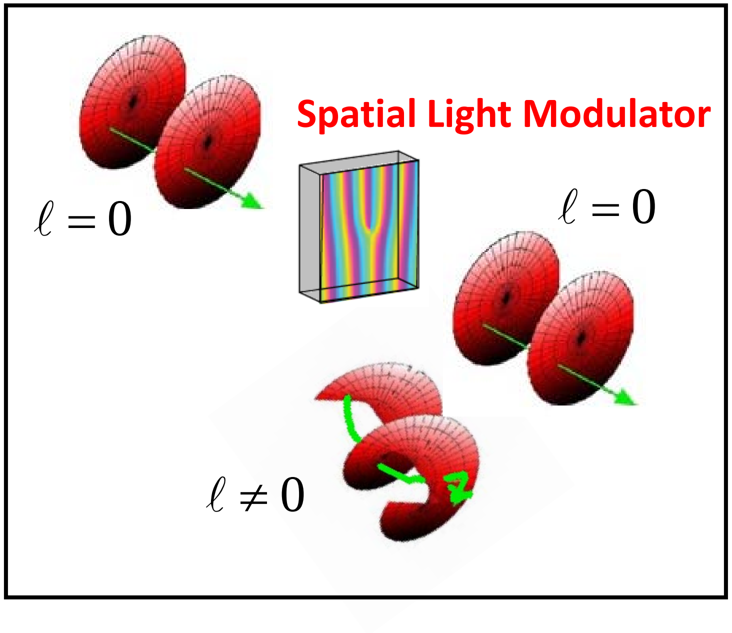
How to generate phase singularities in a controllable manner ?

❖ Spiral phase masks



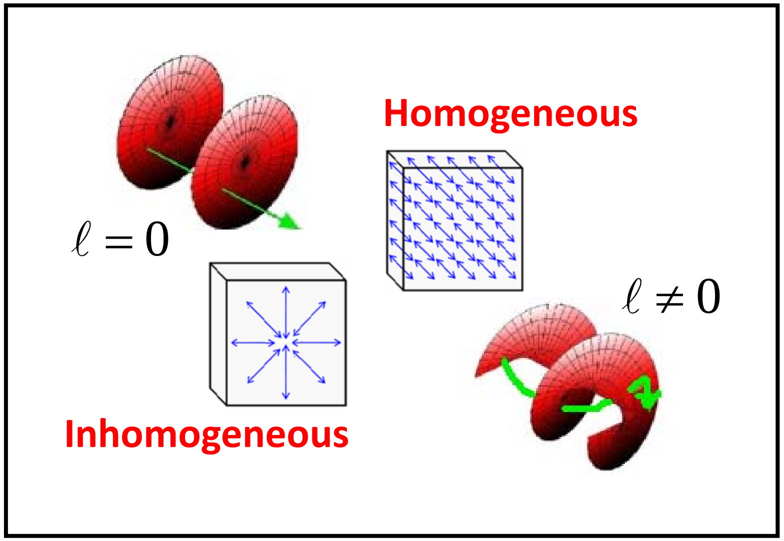
How to generate phase singularities in a controllable manner ?

- ❖ **Spiral phase masks**
- ❖ **Singular gratings**



How to generate phase singularities in a controllable manner ?

- **Spiral phase masks**
- **Singular gratings**
- **Anisotropic media**



Optical spin-orbit interaction of light

Outline

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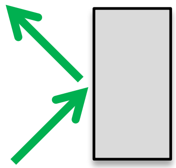
Coupling between the spin of a particle with its motion

Spin angular momentum ⇄ **Orbital angular momentum**

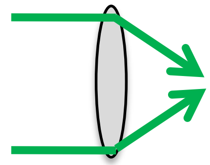
Material inhomogeneity or anisotropy is required

Spin-orbit interaction of light in the lab ?

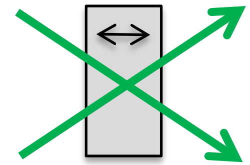
Let us consider simple macroscopic optical elements …



Glass slab

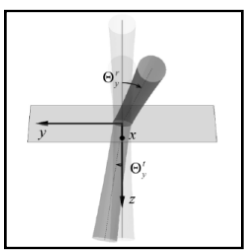


Lens

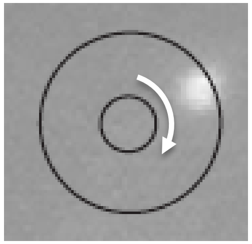


Birefringent crystal slab

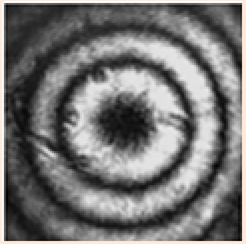
… in the course of circularly polarized light beam



Transverse beam shifts



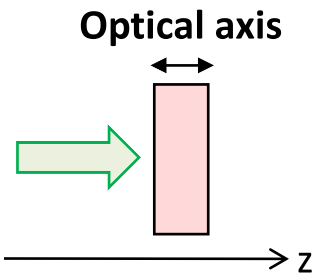
Orbiting motion



Optical vortex

Spin-orbit interaction is around !

Vortex generation using uniaxial crystal optics



Maxwell's equations

$$\boxed{\left(\nabla_\perp^2 + 2ikn_o\partial_z\right)\mathbf{E} = \gamma\nabla_\perp\left(\nabla_\perp\cdot\mathbf{E}\right)}$$

(paraxial approximation, slowly varying transverse envelope)

$$\gamma = 1-(n_o/n_e)^2 \qquad \nabla_\perp \equiv \mathbf{e}_x\,\partial_x + \mathbf{e}_y\,\partial_y$$

Let us consider circular polarization basis : $\mathbf{c}_\pm = (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}$

Let us consider incident Gaussian beam : $\mathbf{E}(r, z=0) = (a\,\mathbf{c}^+ + b\,\mathbf{c}^-)\exp(-r^2/w^2)$

Output light field

$$\boxed{\begin{pmatrix} E^+ \\ E^- \end{pmatrix} = -\frac{i\beta w^2}{z-iz_0}e^{\frac{i\beta r^2}{z-iz_0}}\begin{pmatrix} \cos\delta & -ie^{-2i\varphi}\sin\delta \\ -ie^{2i\varphi}\sin\delta & \cos\delta \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}} \qquad \boxed{\varepsilon = (n_o-n_e)/n \ll 1}$$

$$\begin{aligned} \beta &= kn/2 \\ n &= (n_o+n_e)/2 \\ z_0 &= \beta w^2 \end{aligned}$$

$$\boxed{\delta = \frac{\varepsilon\beta r^2\,z}{(z-iz_0)^2}} \longleftrightarrow$$ **Birefringent phase retardation**

Vortex generation using uniaxial crystal optics

General output light field

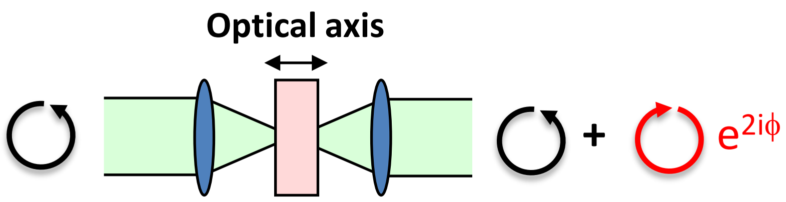
$$\begin{pmatrix} E^+ \\ E^- \end{pmatrix} = -\frac{i\beta w^2}{z - iz_0} e^{\frac{i\beta r^2}{z-iz_0}} \begin{pmatrix} \cos\delta & -ie^{-2i\varphi}\sin\delta \\ -ie^{2i\varphi}\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

Particular case of right-handed circular input : a=1 , b=0

Polarization state changes

$$\mathbf{E} \propto \cos\delta \boxed{\mathbf{c}_+} - i\boxed{e^{2i\varphi}}\sin\delta \boxed{\mathbf{c}_-}$$

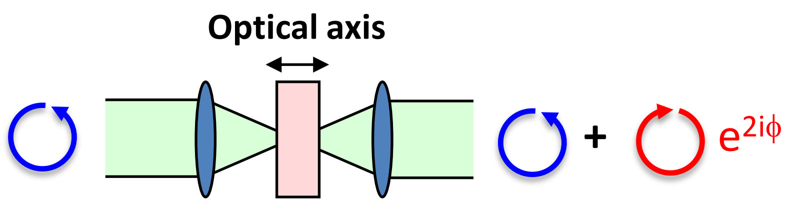
Optical phase singularity birth

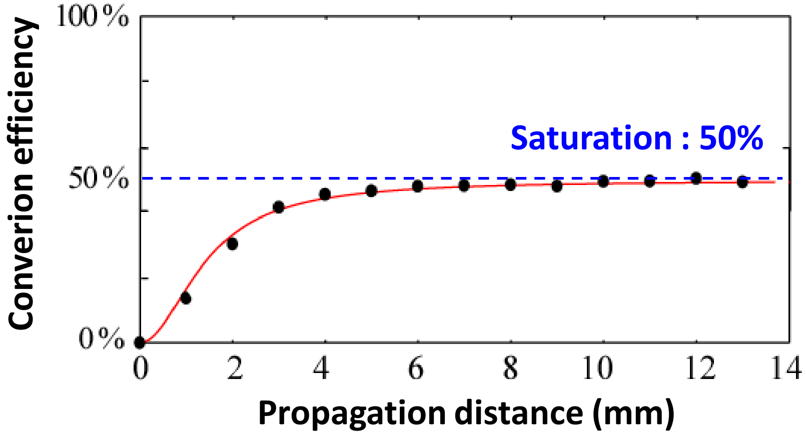


Spin-to-orbital angular momentum conversion

Is it efficient ?

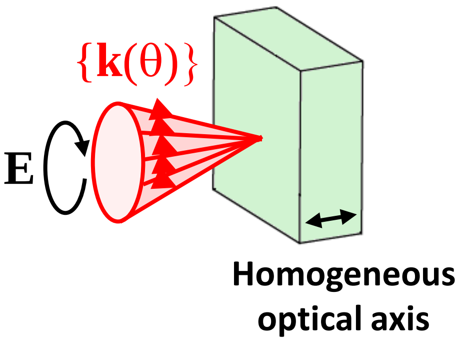
Optical vortex generation in homogeneous uniaxial solid crystals



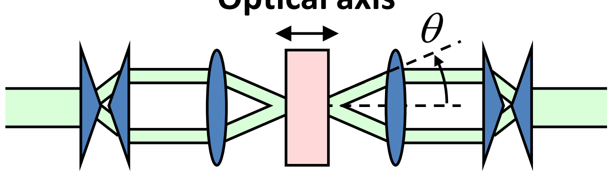


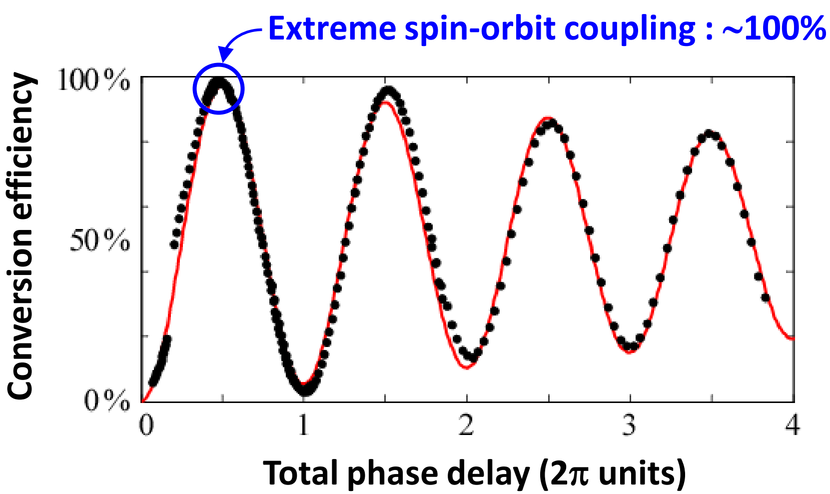
E. Brasselet *et al.*, Opt. Lett. **34**, 1021 (2009)

Beam shaping optimization



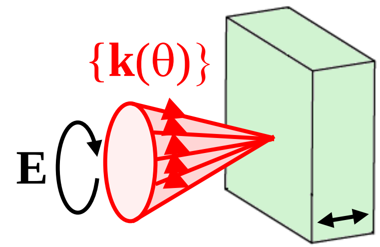
Optical vortex generation in homogeneous uniaxial solid crystals





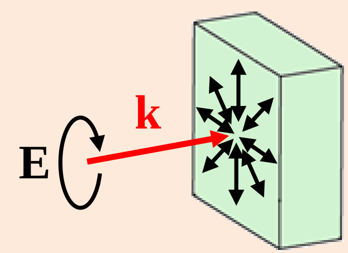
C. Loussert and E. Brasselet, Opt. Lett. **35**, 7 (2010)

Beam shaping optimization



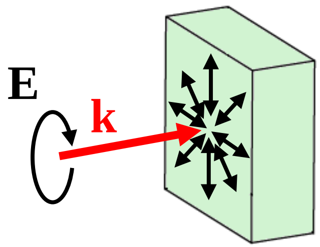
Homogeneous optical axis

Effective material defect



Radially distributed optical axis

Optical vortex generation in inhomogeneous uniaxial medium



Uniform birefringent retardation

$$\Delta = \frac{2\pi}{\lambda}\,\delta n\, L$$

Circularly polarized input field : $\mathbf{E}_{\rm in} = E_0\, e^{-i(\omega t - k_0 z)}\, \mathbf{c}_\sigma \qquad (\sigma = \pm 1)$

Polarization state changes

Output field : $\mathbf{E}_{\rm out} \propto E_0 \left[\cos\left(\Delta/2\right) \boxed{\mathbf{c}_\sigma} + i \sin\left(\Delta/2\right) \boxed{e^{i2\sigma\phi}} \boxed{\mathbf{c}_{-\sigma}}\right]$

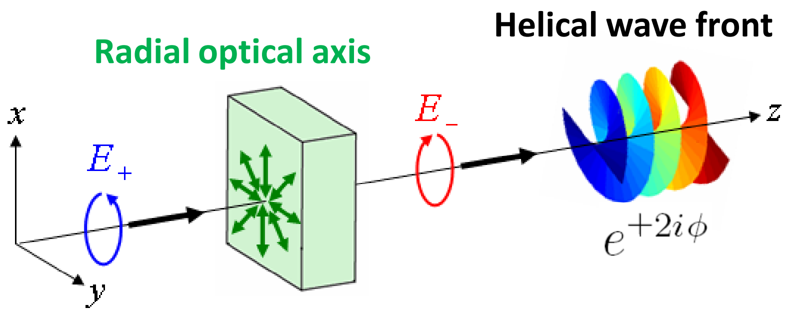
Optical phase singularity birth

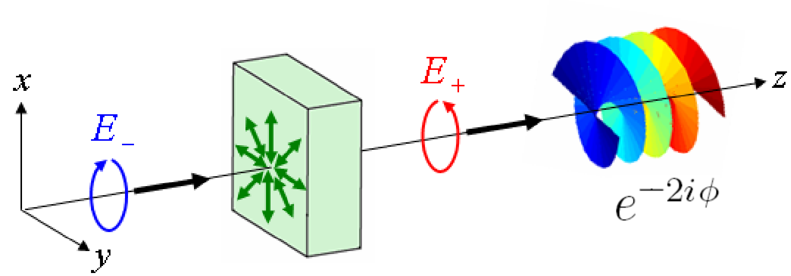
Spin-to-orbital angular momentum conversion

100% efficient when Δ = π

Optical vortex generation in inhomogeneous uniaxial medium

Optimal conversion $\Delta = \pi$



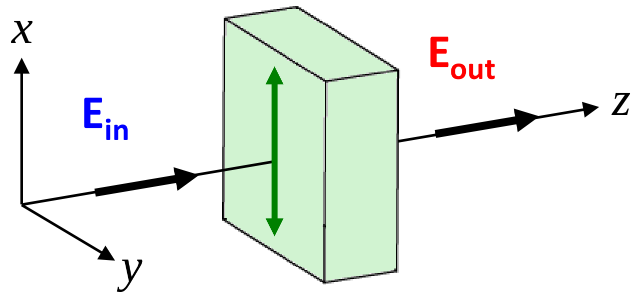


Angular momentum per photon ($\hbar$)			
	Spin	Orbital	Total
Input	1	0	1
Output	– 1	2	1

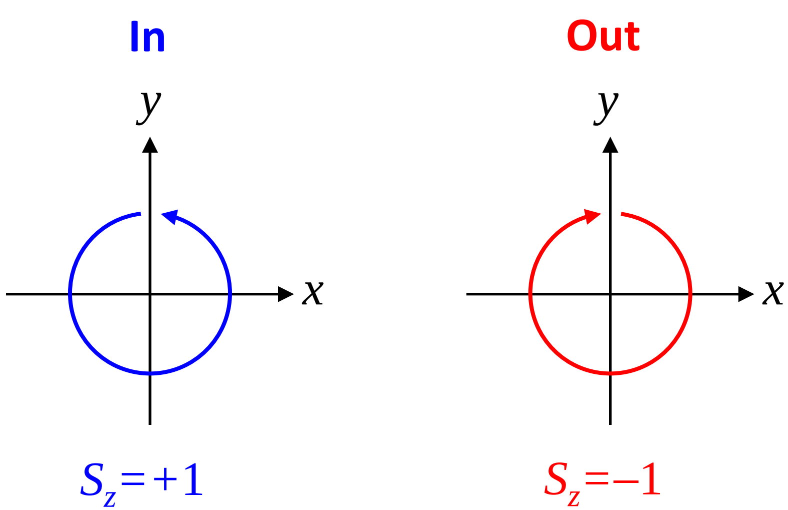
Angular momentum per photon ($\hbar$)			
	Spin	Orbital	Total
Input	– 1	0	– 1
Output	1	– 2	– 1

Conservation of total (spin+orbital) optical angular momentum

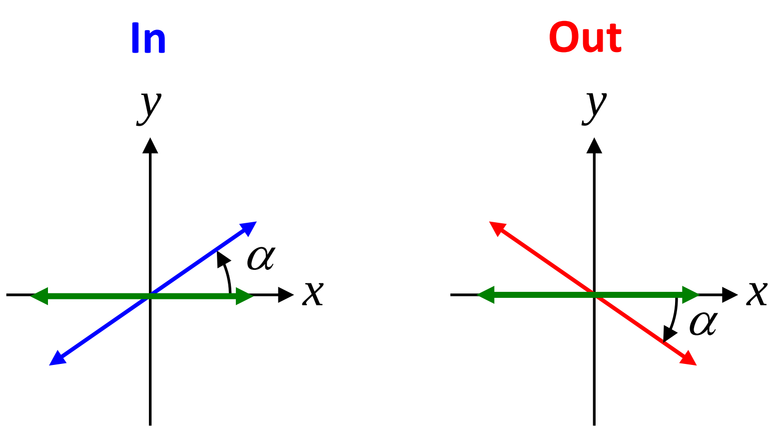
Qualitative analysis



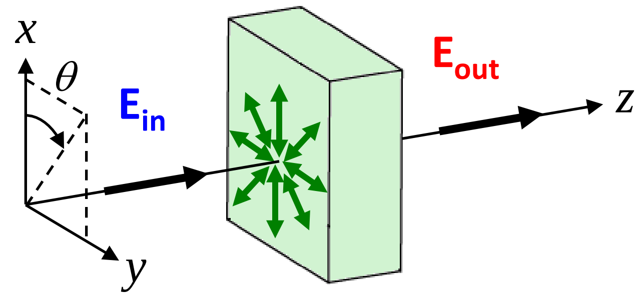
Circular polarization case



Linear polarization case



Qualitative analysis

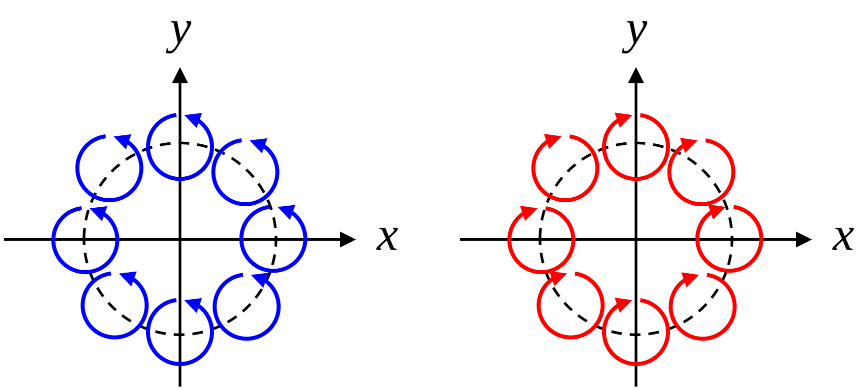


Optical axis radial distribution

At a given time

Circular polarization case

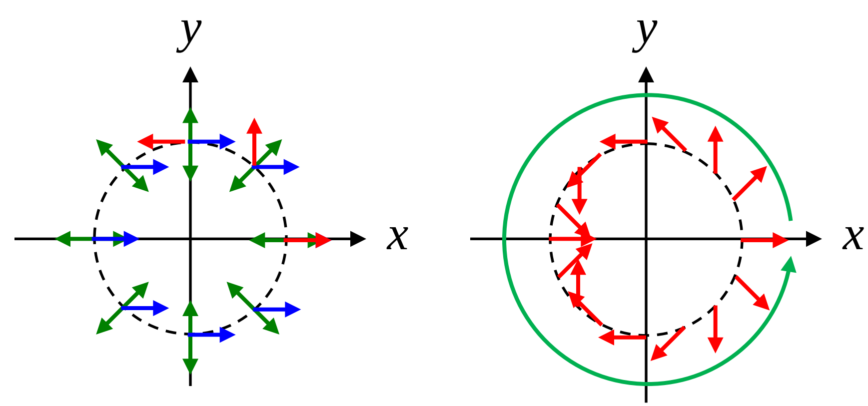
In **Out**



$S_z = +1$ $S_z = -1$

« Linear polarization case»

In **Out**



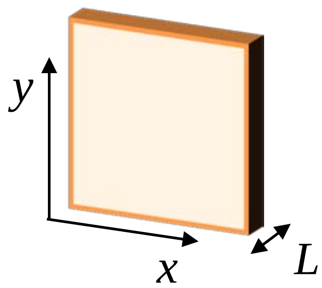
Phase winds by 4π

Generalization to azimuthally patterned birefringent elements

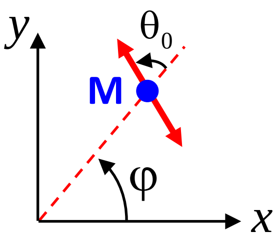
Azimuthally patterned optical axis

$$\theta = m\varphi + \theta_0$$

$$(m \in \mathbb{Z} \ \text{ or } \ 2m \in \mathbb{Z})$$



Local optical axis direction



Local phase delay

$$\Psi(\mathrm{M}) = \frac{2\pi}{\lambda}\Delta n(\mathrm{M})\,L$$

Circularly polarized incident plane wave : $\mathbf{E}_{\rm in} = E_0\,\mathbf{c}_\pm = E_0\,\frac{\mathbf{e}_x \pm i\mathbf{e}_y}{\sqrt{2}}$

Polarization state changes

Output field : $\mathbf{E}_{\rm out} \propto E_0\left[\cos(\Psi/2)\,\boxed{\mathbf{c}_\pm} \mp i\,e^{\pm i2m(\phi + \theta_0)}\sin(\Psi/2)\,\boxed{\mathbf{c}_\mp}\right]$

Phase singularity with topological charge ±2*m*

Generalization to azimuthally patterned birefringent elements

Optical angular momentum balance

$$\mathbf{E}_{\mathrm{out}} \propto E_0 \left[\cos(\Psi/2)\,\mathbf{c}_{\pm} \mp i\, e^{\pm i2m(\phi+\theta_0)} \sin(\Psi/2)\,\mathbf{c}_{\mp} \right]$$

Angular momentum per photon ($\hbar$)			
	Spin	Orbital	Total
Input	1	0	1
Output	–1	2m	2m – 1

Angular momentum per photon ($\hbar$)			
	Spin	Orbital	Total
Input	– 1	0	– 1
Output	1	– 2m	1 – 2m

Total optical angular momentum is no longer preserved for m≠1

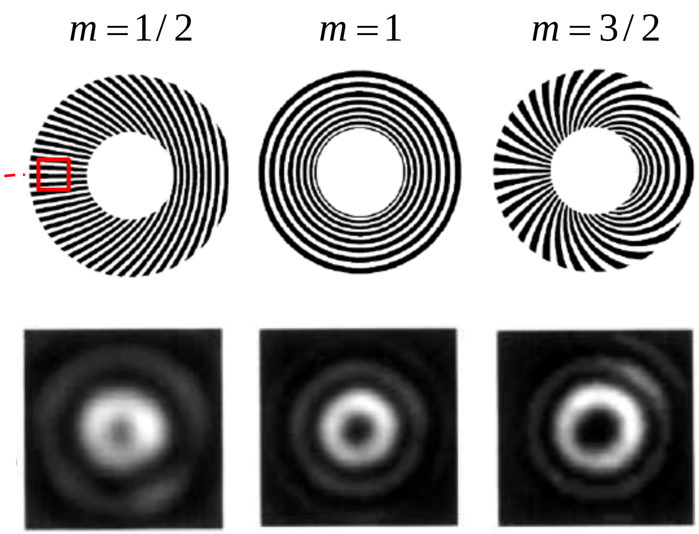
⇩

A torque is exerted on the optical element for m≠1

Outline

1. Introduction to singular optics
2. Optical vortex generation
3. Spin-orbit interaction of light
4. **Spin-to-orbital angular momentum converters**
5. Towards integrated spin-orbit optical vortex generators

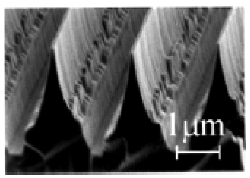
Initially demonstrated at 10.6μm wavelength



Subwavelength gratings

(FORM birefringence)

Optical vortex generation



Laser-etched GaAs wafers

G. Biener *et al*., Opt. Lett. **27**, 1875 (2002)

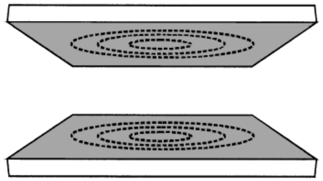
Extended to the visible domain : mechanically patterned liquid crystals

L. Marrucci *et al*., Phys. Rev. Lett. **96**, 163905 (2006)

(TRUE birefringence)

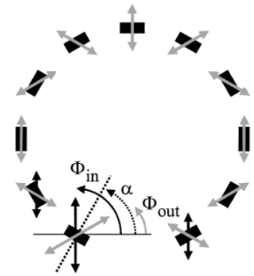
Azimuthally patterned nematic liquid crystal films

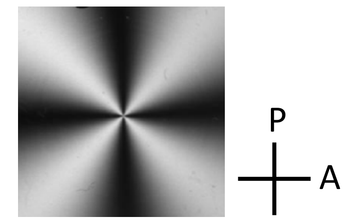
Mechanically prepared
« circular alignment»



Half-wave plate condition
(Δ=π)

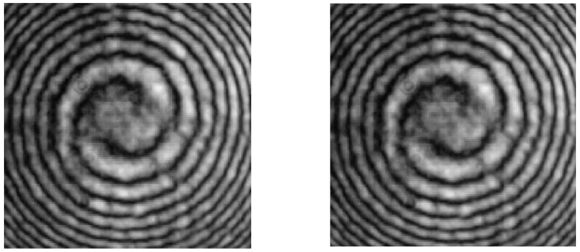
Incident linear polarization
↓
Vector beam





M. Stadler and Schadt, Opt. Lett. **21**, 1948 (1996)

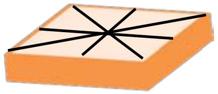
Incident circular polarization
↓
Scalar vortex beam



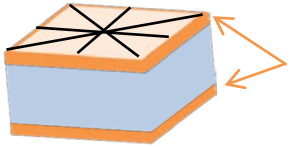
L. Marrucci *et al.*, PRL **96**, 163905 (2006)

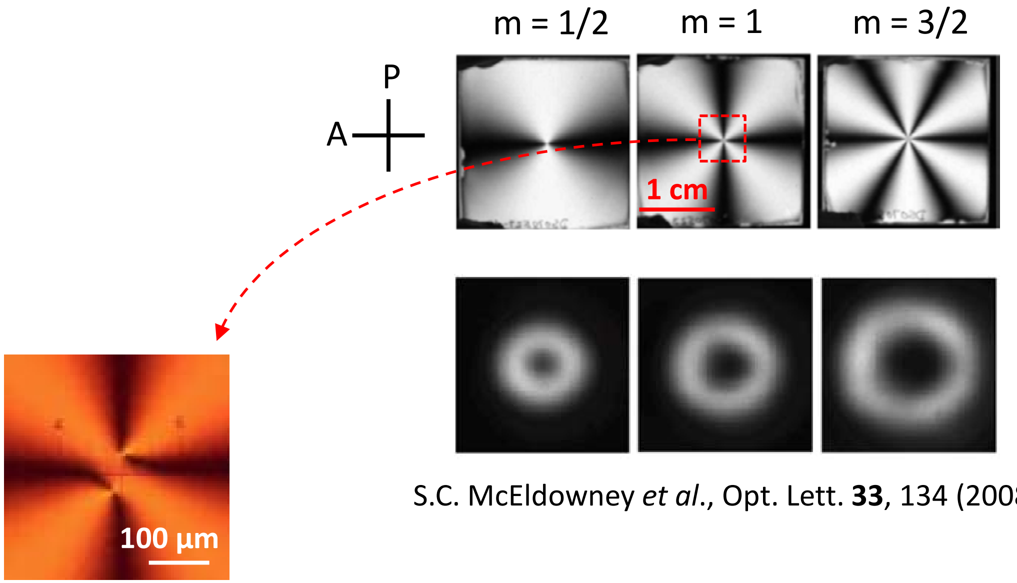
Photo-alignment techniques : patterned liquid crystal films

Bulk approach
- S. C. McEldowney *et al.*, Opt. Lett. **33**, 134 (2008)
- S. Nersisyan *et al.*, Opt. Express **17**, 11926 (2009)



Surface approach
- S. W. Ko *et al.*, Opt. Express **16**, 19643 (2008)
- S. Slussarenko *et al.*, Opt. Express **19**, 4087 (2011)





S.C. McEldowney *et al.*, Opt. Lett. **33**, 134 (2008)

D. Mawet *et al.*, Opt. Express **17**, 1902 (2009)

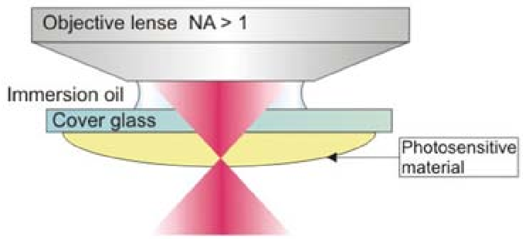
Topology is not controlled at microscale

Outline

1. Introduction to singular optics
2. Optical vortex generation
3. Spin-orbit interaction of light
4. Liquid crystal spin-to-orbital angular momentum converters
5. **Towards integrated spin-orbit optical vortex generators**

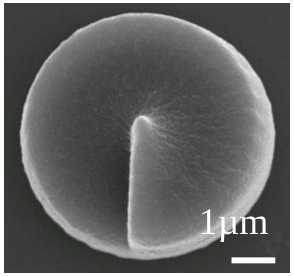
Microscopic spiral phase plates : fabrication

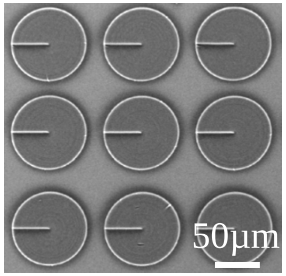
Direct laser writing : on-demand photopolymerized 3D structures



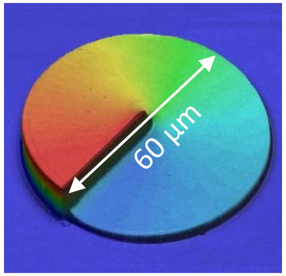
Tightly focused pulsed light

Single or arrays of spiral plates

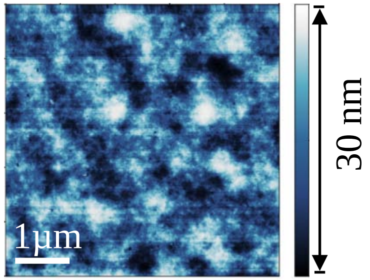




Optical quality spiral plate

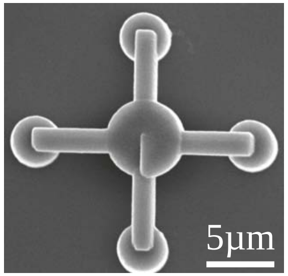


Optical profilometry



AFM image

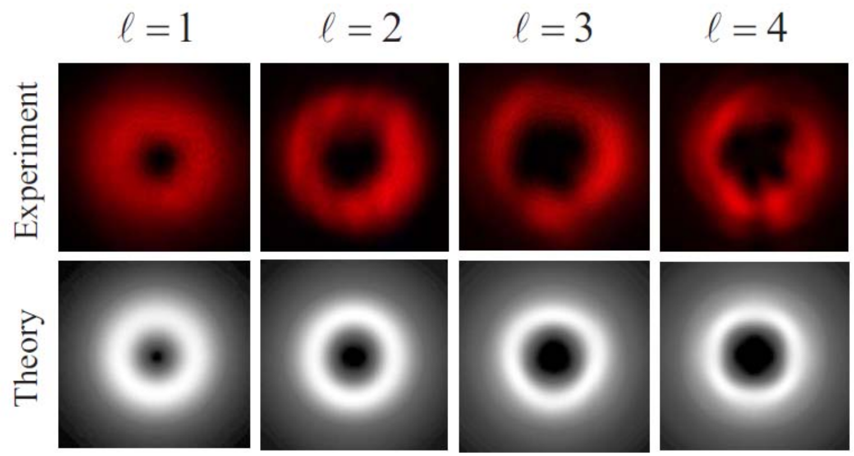
3D micro-architectures

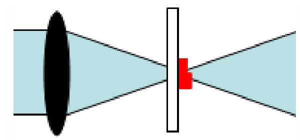


E. Brasselet *et al.*, Appl. Phys. Lett. **97**, 211108 (2010)

Microscopic spiral phase plates : performance characterization

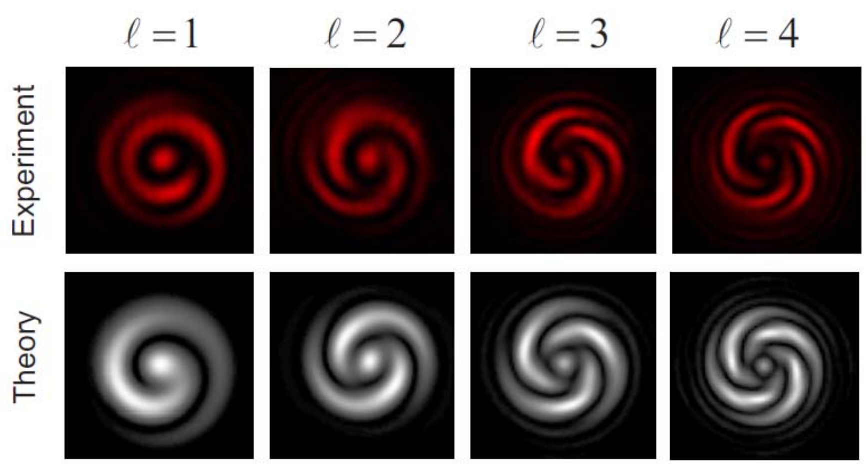
Amplitude

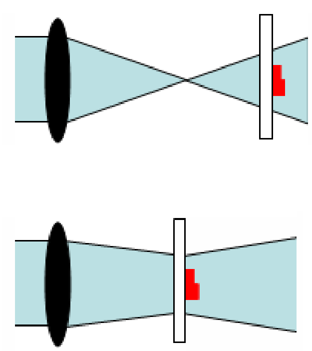




w_0 = R/2

Phase





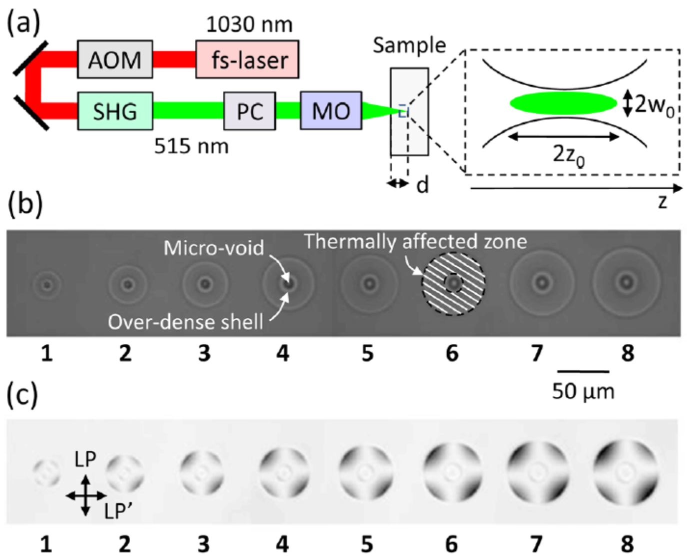
w_0 = 2R

Single-beam interferences technique

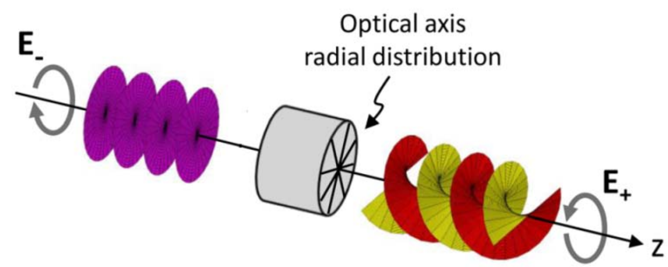
E. Brasselet *et al.*, Appl. Phys. Lett. **97**, 211108 (2010)

Microscopic spin-to-orbital angular momentum converters

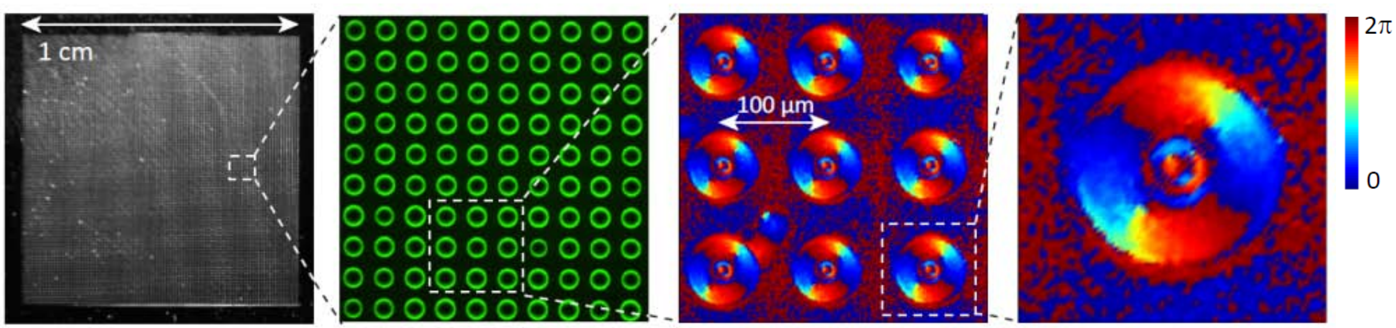
Direct laser writing of radial birefringence



Spin-to-orbital optical angular momentum conversion



Large and dense arrays of optical vortex generators : $10^4/cm^2$

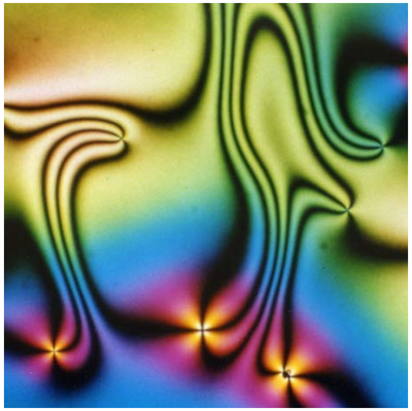


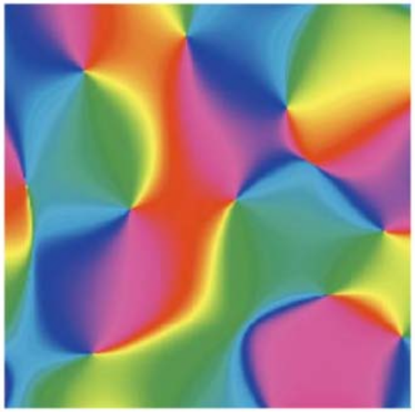
E. Brasselet *et al.*, Appl. Phys. Lett. **100**, 181901 (2012)

Conclusion : topological interplay between matter and light

Imprinting material topological information on light

Liquid crystal defects





Optical phase singularities

Imprinting optical topological information on matter